Static quantities of the W boson in the $SU_L(3)\times U_X(1)$ model with right-handed neutrinos

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The static electromagnetic properties of the W boson, $\Delta\kappa$ and ΔQ, are calculated in the $SU_L(3)\times U_X(1)$ model with right-handed neutrinos. The new contributions from this model arise from the gauge and scalar sectors. In the gauge sector there is a new contribution from a complex neutral gauge boson Y^0 and a singly charged gauge boson $Y^{\pm}$. The mass of these gauge bosons, called bileptons, is expected to be in the range of a few hundred GeV according to the current bounds from experimental data. If the bilepton masses are of the order of 200 GeV, the size of their contribution is similar to that obtained in other weakly coupled theories. However, the contributions to both ΔQ and $\Delta\kappa$ are negligible for very heavy or degenerate bileptons. As for the scalar sector, a scenario is examined in which the contribution to the W form factors is identical to that of a two-Higgs-doublet model. It is found that this sector would not give large corrections to $\Delta\kappa$ and ΔQ.

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I. INTRODUCTION

The experimental scrutiny of the Yang-Mills sector is essential to test the standard model (SM). In particular, the trilinear gauge boson couplings (TGBCs) offer a unique opportunity to find evidence of new physics through the study of their one-loop corrections. Hopefully, the TGBCs will be tested with unprecedent accuracy beyond the tree level at hadronic and leptonic colliders in the near future [1]. Particular emphasis has been given to the study of the static quantities of the W boson. The CP-even electromagnetic properties of the W boson are characterized by two form factors, $\Delta\kappa$ and ΔQ, which are the coefficients of Lorentz tensors of canonical dimension 4 and 6, respectively [2]. Both form factors can only arise at the one-loop level within the SM and other renormalizable theories, thereby being sensitive to sizable new physics effects. It has been argued that ΔQ is not sensitive to nondecoupling effects and thus it could only be useful to search for effects of new physics near the Fermi scale [3]. On the contrary, $\Delta\kappa$ might be sensitive to heavy physics effects due to its nondecoupling properties [3]. The one-loop contributions to ΔQ and $\Delta\kappa$ were long ago studied in the SM [4,5] and more recently in the context of several specific theories [6–8]. Also, a model-independent study of the $WW\gamma$ vertex via the effective Lagrangian approach was presented in Ref. [9].

In a recent work [8] two of us studied the static quantities of the W boson in the context of the minimal $SU_L(3)\times U_X(1)$ model, dubbed the 3-3-1 model [10]. The main attraction of this model is the unique mechanism of anomaly cancellation, which is achieved provided that all of the fermion families are summed over rather than within each fermion family, as occurs in the SM. As a consequence, the number of fermion families must be a multiple of the quark color number, which offers a possible solution to the flavor problem. The 3-3-1 model has been the source of considerable interest recently [11]. In this work we will focus on the contributions to $\Delta\kappa$ and ΔQ from both the gauge and scalar sectors of the 3-3-1 model with right-handed neutrinos [12]. This version is attractive because, in order to achieve the mechanism of spontaneous symmetry breaking (SSB) and generate the gauge boson and fermion masses, it requires a Higgs sector which is more economic than that of the minimal version [12]. Evidently the features of the 3-3-1 model with right-handed neutrinos are rather different than those of the minimal version, and so are the contributions to the static quantities of the W boson. It is thus worth evaluating the behavior of $\Delta\kappa$ and ΔQ in the new scenario raised by this model. Special attention will be paid to discuss the contribution arising from the gauge sector because it is the one which has the more interesting features. As will be shown below, the contribution from the scalar sector is similar to that arising in a two-Higgs-doublet model (THDM) [6].

A peculiarity of 3-3-1 models is that they predict a pair of massive gauge bosons arranged in a doublet of the electroweak group, which emerge when $SU_L(3)\times U_X(1)$ is broken into $SU_L(2)\times U_Y(1)$. While the minimal 3-3-1 model predicts a pair of singly charged and a pair of doubly charged gauge bosons, the model with right-handed neutrinos predicts a pair of neutral no-self-conjugate gauge bosons Y^0 instead of the doubly charged ones. These new gauge bosons are called bileptons since they carry lepton number $L=\pm 2$, and thus are responsible for lepton-number violating interactions [13]. It has been pointed out that the neutral bilepton is a promising candidate in accelerator experiments since it may be the source of neutrino oscillations [14]. Very interestingly, the couplings between the SM gauge bosons and the bileptons do not involve any mixing angle and are similar in strength to the couplings existing between the SM gauge

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TABLE I. Fermion spectrum of the 3-3-1 model with right-handed neutrinos, along with the quantum number assignments.

Leptons	First two-quark families	Third-quark family
$f_L^i=\begin{pmatrix}\nu_L^i\\ e_L^i\\ (\nu_L^c)^i\end{pmatrix}\sim(1,3,-1/3)$	$Q_L^i=\begin{pmatrix}d_L^i\\ -u_L^i\\ D_L^i\end{pmatrix}\sim(3,\overline{3},0)$	$Q_L^3=\begin{pmatrix}u_L^3\\ d_L^3\\ T_L\end{pmatrix}\sim(3,3,1/3)$
$e_R^i\sim(1,1,-1)$	$u_R^i\sim(3,1,2/3)$	$u_R^3\sim(3,1,2/3)$
	$d_R^i\sim(3,1,-1/3)$	$d_R^3\sim(3,1,-1/3)$
	$D_R^i\sim(3,1,-1/3)$	$T_R\sim(3,1,2/3)$

bosons themselves. Current bounds establish that the bilepton masses may be in the range of a few hundred GeV [15]. It is then feasible that these bileptons may show up through their virtual effects in low-energy processes. This is an important reason to investigate the effect of these particles on the $WW\gamma$ vertex function. It is also interesting that, due to the SSB hierarchy, the splitting between the bilepton masses $m_{Y^\pm}$ and m_{Y^0} is bounded by m_W, so the bileptons would be almost degenerate since their masses are expected to be heavier than the W mass. Therefore, the gauge boson contribution to the static quantities of the W boson would depend on only one free parameter. As far as the scalar sector is concerned, this model predicts the existence of ten physical scalar bosons [16]: four neutral *CP*-even, two neutral *CP*-odd, and four charged ones. From these scalar bosons, only three of them, two neutral *CP*-even and a charged one, couple with the W boson at the tree level because they are the only ones which get their masses at the Fermi scale. Furthermore, in order to reproduce the SM at low energies, we will concentrate on a scenario in which one of the neutral Higgs bosons coincides with the SM Higgs boson.

The rest of the paper is organized as follows. Section II is devoted to a brief description of the 3-3-1 model with right-handed neutrinos. In Sec. III we present the calculation of the static properties of the W boson. The numerical results are analyzed in Sec. IV, and the conclusions are presented in Sec. V.

II. BRIEF REVIEW OF THE 3-3-1 MODEL WITH RIGHT-HANDED NEUTRINOS

The features of the 3-3-1 model with right-handed neutrinos has been discussed to a large extent in Ref. [12]. Here we will only review those aspects which are relevant for the present discussion. The fermion spectrum of the model is shown in Table I. The three lepton families are arranged in triplets of $SU_L(3)$, whereas in the quark sector it is necessary to introduce three exotic quarks D_1, D_2, and T. In order to cancel the $SU_L(3)$ anomaly, two quark families must transform as $SU_L(3)$ antitriplets and the remaining one as a triplet. It is customary to arrange the first two-quark families in antitriplets and the third one in a triplet. This choice is meant to distinguish the possible new dynamics effects arising in the third family.

The electric charges of the exotic quarks are $Q_{D_1D_2}=-1/3e$ and $Q_T=2/3e$. This is to be contrasted with the three new quarks, D, S, and T, predicted by the minimal 3-3-1 model, whose charge is indeed exotic, namely $Q_{D,S}=-4/3e$ and $Q_T=5/3e$.

As already mentioned, the 3-3-1 model with right-handed neutrinos has the advantage that it requires a Higgs sector simpler than that introduced in the minimal version. In fact, only three triplets of $SU_L(3)$ are needed to reproduce the known physics at the Fermi scale:

$$\chi=\begin{pmatrix}\Phi_3\\ \\ \chi'^0\end{pmatrix}\sim(1,3,-1/3),\quad \rho=\begin{pmatrix}\Phi_1\\ \\ \rho'^+\end{pmatrix}\sim(1,3,2/3),$$

$$\eta=\begin{pmatrix}\Phi_2\\ \\ \eta'^0\end{pmatrix}\sim(1,3,-1/3), \tag{1}$$

where $\Phi_1^\dagger=(\rho^-,\rho^{0*})$, $\Phi_2^\dagger=(\eta^{0*},\eta^+)$, and $\Phi_3^\dagger=(\chi'^{0*},\chi^+)$ are $SU_L(2)\times U_Y(1)$ doublets with hypercharge $+1$, -1, and -1, respectively. This is to be contrasted again with the minimal 3-3-1 model, which requires the presence of three triplets and one sextet [10]. The vacuum expectation value (VEV) $\langle\chi\rangle^T=(0,0,w/\sqrt{2})$ breaks down the $SU_L(3)\times U_N(1)$ group into $SU_L(2)\times U_Y(1)$. In this first stage of SSB, the new gauge bosons and quarks, as well as some physical scalars, get their masses. At the Fermi scale all the known SM particles and some physical scalar bosons are endowed with masses through the VEV $\langle\Phi_1\rangle=(0,v_1/\sqrt{2})$ and $\langle\Phi_1\rangle=(0,v_1/\sqrt{2})$. In this way, the Φ_1 and Φ_2 doublets break the $SU_L(2)\times U_Y(1)$ group into $U_e(1)$.

In addition to the three exotic quarks, the model predicts the existence of five new gauge bosons: two singly charged $Y^\pm$, two neutral no-self-conjugate Y^0, and a neutral self-conjugate Z'. The $Y^\pm$ and Y^0 gauge bosons are called bileptons because they carry two units of lepton number [13]. All the new particles acquire their masses at the w scale. At this stage of SSB, the exotic quarks together with the Z' boson emerge as singlets of the electroweak group. Consequently, these particles cannot interact with the W boson at the tree level. It follows that there is no contribution from the exotic quarks to the static electromagnetic properties of the W boson at the lowest order. As for the Z' boson, it couples to the W boson via the Z'-Z mixing induced at the Fermi scale, which means that the respective contribution to the $WW\gamma$ vertex is expected to be strongly suppressed since it is proportional to the corresponding mixing angle, which is expected to be negligibly small [12]. On the other hand, the dynamical behavior of the bileptons is different since they arise as a doublet of the electroweak group at the w scale and thus have nontrivial couplings with the SM gauge bosons. Due to the fact that the $SU_L(2)$ group is completely embedded in $SU_L(3)$, the bileptons couple with the SM gauge bosons with a strength similar to that of the couplings existing between the SM gauge bosons. In particular, these new couplings are entirely determined by the coupling constant associated with $SU_L(2)$ and the weak angle θ_W. When $SU_L(2)\times U_Y(1)$ is broken down to $U_e(1)$, the masses of the bileptons receive new contributions. In the gauge sector, the mass eigenstates arise from the Higgs kinetic-energy term, which is given by

$$\mathcal{L}=(D_\mu\chi)^\dagger(D^\mu\chi)+(D_\mu\rho)^\dagger(D^\mu\rho)+(D_\mu\eta)^\dagger(D^\mu\eta), \tag{2}$$

where D_μ is the covariant derivative associated with the $SU_L(3)\times U_N(1)$ group, which in the fundamental representation is given by

$$D_\mu=\partial_\mu-g\frac{\lambda^a}{2}A_\mu^a-ig_N\frac{\lambda^9}{2}N_\mu, \tag{3}$$

with λ^a $(a=1,2,\ldots,8)$ being the Gell-Mann matrices and $\lambda^9=\sqrt{2/3}\mathrm{diag}(1,1,1)$. Once this sector is diagonalized, there emerge the following mass-eigenstate fields:

$$Y_\mu^0=\frac{1}{\sqrt{2}}(A_\mu^4-iA_\mu^5), \tag{4}$$

$$Y_\mu^-=\frac{1}{\sqrt{2}}(A_\mu^6-iA_\mu^7), \tag{5}$$

$$W_\mu^+=\frac{1}{\sqrt{2}}(A_\mu^1-iA_\mu^2), \tag{6}$$

with masses $m_{Y^0}^2=g^2(w^2+v_2^2)/4$, $m_{Y^\pm}^2=g^2(w^2+v_1^2)/4$, and $m_W^2=g^2(v_1^2+v_2^2)/4$. From these expressions, it is easy to see that there is an upper bound on the splitting between the bilepton masses:

$$|m_{Y^0}^2-m_{Y^\pm}^2|\leq m_W^2. \tag{7}$$

The remaining three gauge fields A_μ^3, A_μ^8, and N_μ define the self-conjugate mass eigenstates A_μ, Z_μ, and Z'_μ [12]. As far as the Yang-Mills sector of the model is concerned, it is given by

$$\mathcal{L}_{YM}=-\frac{1}{4}F_{\mu\nu}^aF_a^{\mu\nu}-\frac{1}{2}N_{\mu\nu}N^{\mu\nu}, \tag{8}$$

where $F_{\mu\nu}^a=\partial_\mu A_\nu^a-\partial_\nu A_\mu^a+f^{abc}A_\mu^bA_\nu^c$ and $N_{\mu\nu}=\partial_\mu N_\nu-\partial_\nu N_\mu$, being f^{abc} the structure constants associated with $SU_L(3)$. After this Lagrangian is expressed in terms of mass-eigenstate fields, it can be split into three $SU_L(2)\times U_Y(1)$-invariant pieces:

$$\mathcal{L}_{YM}=\mathcal{L}_{YM}^{SM}+\mathcal{L}_{YM}^{SM\text{-}NP}+\mathcal{L}_{YM}^{NP}, \tag{9}$$

where the first term represents the well-known Yang-Mills sector associated with the electroweak group, whereas $\mathcal{L}_{YM}^{SM\text{-}NP}$ represents the interactions between the SM gauge fields and the heavy ones:

$$\begin{aligned}\mathcal{L}_{YM}^{SM\text{-}NP}=&-\frac{1}{2}(D_\mu Y_\nu-D_\nu Y_\mu)^\dagger(D^\mu Y^\nu-D^\nu Y^\mu)\\&-iY_\mu^\dagger(g\mathbf{F}^{\mu\nu}+g'\mathbf{B}^{\mu\nu})Y_\nu\\&-\frac{ig}{2}\frac{\sqrt{3-4s_W^2}}{c_W}Z'_\mu[Y_\nu^\dagger(D^\mu Y^\nu-D^\nu Y^\mu)\\&-(D^\mu Y^\nu-D^\nu Y^\mu)^\dagger Y_\nu],\end{aligned} \tag{10}$$

where $Y_\mu^\dagger=(Y_\mu^{0*},Y_\mu^+)$ is a doublet of the electroweak group with hypercharge -1, and $D_\mu=\partial_\mu-ig\mathbf{A}_\mu+ig'\mathbf{B}_\mu$ is the covariant derivative associated with this group. We have introduced the definitions $\mathbf{F}_{\mu\nu}=\sigma^iF_{\mu\nu}^i/2$, $\mathbf{A}_\mu=\sigma^iA_\mu^i/2$, and $\mathbf{B}_\mu=YB_\mu/2$, with σ^i the Pauli matrices. Finally, the last term in Eq. (9) is also invariant under the electroweak group and comprises the interactions between the heavy gauge fields. There are no contributions to the $WW\gamma$ vertex arising from this Lagrangian and we refrain from presenting the respective expression here.

In the unitary gauge, the contributions to the $WW\gamma$ vertex arise from the first two terms of the Lagrangian $\mathcal{L}_{YM}^{SM\text{-}NP}$. These contributions are given by the vertices $W^\pm Y^\mp Y^0$, $W^\pm W^\mp\gamma$, $Y^\pm W^\mp Y^0\gamma$, $W^\pm W^\mp Y^\pm Y^\mp$, and $W^\pm Y^\mp Y^0\gamma$. The corresponding Feynman rules are represented in Fig. 1.

As far as the scalar sector is concerned, it was analyzed in detail in Refs. [12,16]. Although the most general Higgs potential is very cumbersome, it gets simplified to a large extent if one assumes the discrete symmetry $\chi\to-\chi$ [12,16]. Under this assumption, the scalar potential can be written in the following way:

$$\begin{aligned}V(\chi,\rho,\eta)=&\mu_1^2(\eta^\dagger\eta)+\mu_2^2(\rho^\dagger\rho)+\mu_3^2(\chi^\dagger\chi)\\&+\lambda_1(\eta^\dagger\eta)^2+\lambda_2(\rho^\dagger\rho)^2+\lambda_3(\chi^\dagger\chi)^2\\&+(\eta^\dagger\eta)[\lambda_4(\rho^\dagger\rho)+\lambda_5(\chi^\dagger\chi)]\\&+\lambda_6(\rho^\dagger\rho)(\chi^\dagger\chi)+\lambda_7(\rho^\dagger\eta)(\eta^\dagger\rho)+\lambda_8(\chi^\dagger\eta)\\&\times(\eta^\dagger\chi)+\lambda_9(\rho^\dagger\chi)(\chi^\dagger\rho)+\lambda_{10}(\chi^\dagger\eta+\eta^\dagger\chi)^2.\end{aligned} \tag{11}$$

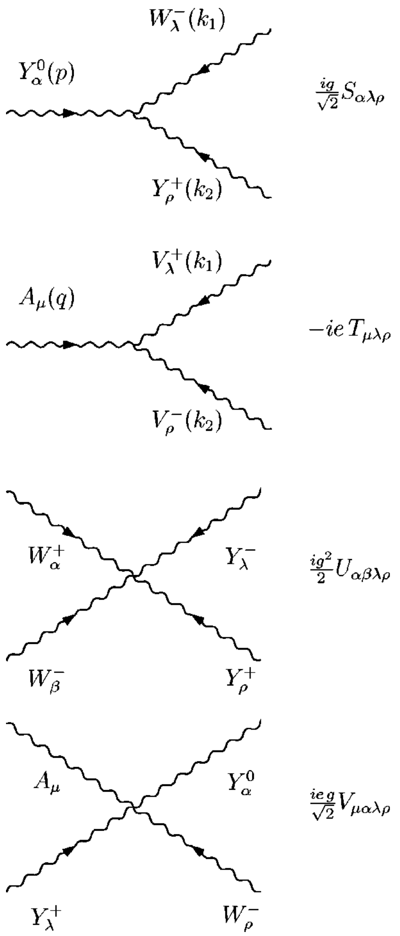

FIG. 1. Unitary gauge Feynman rules for the vertices which contribute to the on-shell $WW\gamma$ vertex in the gauge sector of the 3-3-1 model with right-handed neutrinos. The arrows represent the flow of the 4-momenta. $S_{\alpha\lambda\rho}=(p-k_2)_\lambda g_{\rho\alpha}+(k_2-k_1)_\alpha g_{\lambda\rho}+(k_1-p)_\rho g_{\alpha\lambda}$, $T_{\mu\lambda\rho}=(k_2-k_1)_\mu g_{\lambda\rho}+(q-k_2)_\lambda g_{\mu\rho}+(k_1-q)_\rho g_{\mu\lambda}$, $U_{\alpha\beta\lambda\rho}=2g_{\alpha\rho}g_{\beta\lambda}-g_{\alpha\beta}g_{\lambda\rho}-g_{\beta\rho}g_{\alpha\lambda}$, and $V_{\mu\alpha\lambda\rho}=g_{\alpha\lambda}g_{\rho\mu}-2g_{\alpha\mu}g_{\lambda\rho}+g_{\alpha\rho}g_{\lambda\mu}$.

It is worthwhile to analyze the behavior of the scalar potential at the first stage of SSB. To this end we split $V(\chi,\rho,\eta)$ into the following two terms:

$$V(\chi,\rho,\eta)=V(\Phi_1,\Phi_2)+V_w, \tag{12}$$

with

$$\begin{aligned}V(\Phi_1,\Phi_2)=&\mu_1^2(\Phi_2^\dagger\Phi_2)+\mu_2^2(\Phi_1^\dagger\Phi_1)+\lambda_1(\Phi_2^\dagger\Phi_2)^2\\&+\lambda_2(\Phi_1^\dagger\Phi_1)^2+\lambda_4(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)\\&+\lambda_7(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1),\end{aligned} \tag{13}$$

and V_w an intricate function which includes all those terms not appearing in $V(\Phi_1,\Phi_2)$. V_w is not relevant for the present discussion, so we will refrain from presenting its explicit form here. We will content ourselves with mentioning that this term generates the heavy Higgs boson masses, i.e., those which are proportional to the w scale, whereas the SM gauge bosons and the remaining physical scalar bosons receive their masses from $V(\Phi_1,\Phi_2)$ at a relatively light scale. Note that $V(\Phi_1,\Phi_2)$ corresponds to the scalar potential of a THDM and so there are five Higgs bosons, which are relatively light. In fact, the explicit diagonalization of $V(\chi,\rho,\eta)$ leads to five light and five heavy scalar bosons [16]. The light scalar bosons are, in the notation of Ref. [16], two neutral *CP*-even Higgs bosons, H_1 and H_2, a pair of charged ones, $H_5^\pm$, and a massless neutral *CP*-odd Higgs boson, A_2. The last one would receive its mass through radiative corrections. As for the heavy Higgs spectrum, it is composed of two neutral *CP*-even scalar bosons, H_3 and H_4', a neutral *CP*-odd one, A_1, and a pair of charged ones, $H_6^\pm$. As pointed out in Ref. [16], the neutral *CP*-even scalar boson H_2 coincides with the SM one provided that $\lambda_4=\lambda_5$. For the purpose of this work, it is enough to consider this scenario. For the sake of clarity, we will only present the expressions which relates the gauge states to the mass eigenstates of the light sector. The real part of the ρ^0 and η^0 neutral components of Φ_1 and Φ_2 define the *CP*-even states H_1 and H_2 via the following rotation:

$$H_1=c_\beta\eta_r^0-s_\beta\rho_r^0, \tag{14}$$

$$H_2=s_\beta\eta_r^0+c_\beta\rho_r^0, \tag{15}$$

where β is defined by $\tan\beta=(v_2/v_1)$, $c_\beta=\cos\beta$, $s_\beta=\sin\beta$, and the subscript r denotes the real part of the field. In the charged sector, the ρ^+ and η^+ components of Φ_1 and Φ_2 define the charged H_5^+ Higgs boson and the pseudo-Goldstone boson associated with the W gauge boson G_W^+:

$$H_5^+=c_\beta\eta^++s_\beta\rho^+, \tag{16}$$

$$G_W^+=-s_\beta\eta^++c_\beta\rho^+. \tag{17}$$

Finally, the imaginary part of ρ^0 and η^0 define the pseudo-Goldstone boson associated with the Z gauge boson G_Z and the massless *CP*-odd scalar boson A_2.

Once the light Higgs mass eigenstates are defined, from the Higgs-kinetic term it is straightforward to obtain those couplings involving the W gauge boson. In order to analyze the behavior of the Higgs sector at the Fermi scale in the scenario with $\lambda_4=\lambda_5$, we will present the full Lagrangian involving the couplings of the W boson to the neutral and charged Higgs bosons. It can be written as

$$\mathcal{L}=\left(m_W^2+gm_WH_2+\frac{g^2}{4}(H_1^2+H_2^2+2H_5^-H_5^+)\right)W_\mu^-W^{+\mu}. \tag{18}$$

There is a similar expression involving the Z boson. It is also interesting to note that there is no trilinear self-coupling of the H_1 Higgs boson. From this Lagrangian it is evident that the couplings of H_2 to the SM gauge bosons are SM-like, which means that it should be identified with the SM Higgs boson. So its contribution to the $WW\gamma$ vertex should be considered as a part of the SM [4] rather than a new physics effect. In fact, the only contribution which can be considered as a new physics effect is that induced by the $H_5^-H_5^+WW$

vertex. The model also induces the trilinear $W^{\pm}H_5^{\mp}H_1$ and quartic $\gamma W^{\pm}H_1H_5^{\mp}$ vertices, which also can contribute to the $WW\gamma$ coupling. The corresponding Lagrangian for these terms can be written as

$$\mathcal{L}=\frac{ig}{2}[W_\mu^+(H_1\partial^\mu H_5^- - H_5^-\partial^\mu H_1) - W_\mu^-(H_1\partial^\mu H_5^+ - H_5^+\partial^\mu H_1)] + \frac{eg}{2}H_1A_\mu(W^{+\mu}H_5^- + W^{-\mu}H_5^+). \tag{19}$$

There is no analogous Lagrangian for H_2, which is in agreement with the fact that H_2 plays the role of the SM Higgs boson. It is worth noting that all of the couplings which contribute to the $WW\gamma$ vertex are determined entirely by the coupling constant g, in contrast with the case of the most general THDM potential, which involves mixing angles. This fact will simplify considerably the analysis of the $\Delta\kappa$ and ΔQ form factors as they will depend only on the H_1 and H_5^+ masses, which resembles the situation arising in the gauge sector, where the form factors depend only on the bilepton masses. In particular, since both H_1 and H_5^+ receive their masses at the Fermi scale, it is also reasonable to analyze the scenario in which they are degenerate. From the above Lagrangians it is straightforward to obtain the Feynman rules necessary for our calculation. For the sake of completeness they are shown in Fig. 2.

III. STATIC QUANTITIES OF THE W BOSON

In the usual notation, the most general *CP*-even on-shell $WW\gamma$ vertex can be written as [4]

$$\Gamma^{\mu\alpha\beta}=ie\left\{A[2p^\mu g^{\alpha\beta}+4(Q^\beta g^{\mu\alpha}-Q^\alpha g^{\mu\beta})] + \Delta\kappa(Q^\beta g^{\mu\alpha}-Q^\alpha g^{\mu\beta}) + \frac{4\Delta Q}{m_W^2}p^\mu Q^\alpha Q^\beta\right\}, \tag{20}$$

where the momenta of the particles are denoted as follows. $(p-Q)_\alpha$ and $(P+Q)_\beta$ are the momenta of the incoming and outgoing W boson, respectively, and $2Q_\mu$ is that of the photon. We have dropped the *CP*-odd terms since they do not arise in the 3-3-1 model with right-handed neutrinos. This class of terms can be generated, for instance, in models in which the W boson couples to both left- and right-handed fermions simultaneously [17]. Both $\Delta\kappa$ and ΔQ vanish at the tree level in the SM, and the one-loop corrections from the fermion, gauge, and scalar sectors are all of the order of α/π [4]. These form factors define the magnetic dipole moment (μ_W) and the electric quadrupole moment (Q_W) of the W boson, which are given by

$$\mu_W=\frac{e}{2m_W}(2+\Delta\kappa), \tag{21}$$

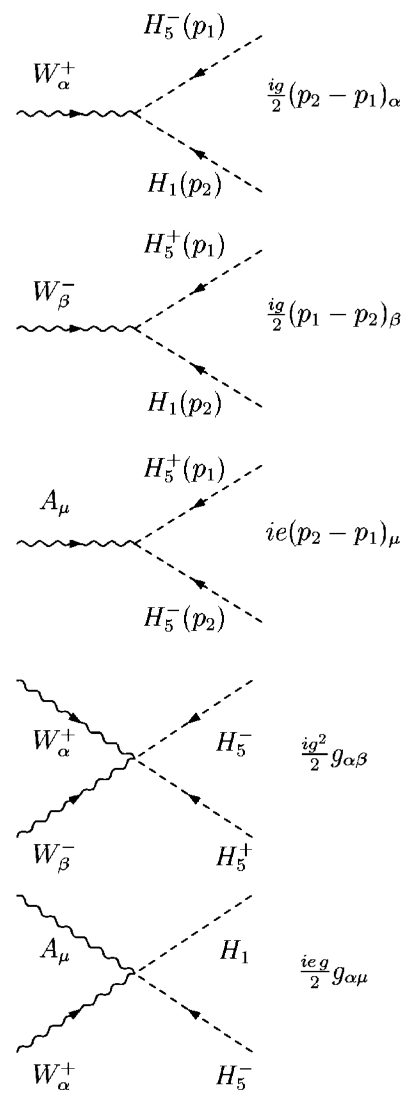

FIG. 2. Feynman rules for the vertices which contribute to the on-shell $WW\gamma$ vertex in the scalar sector of the 3-3-1 model with right-handed neutrinos. The arrows represent the flow of the 4-momenta. The coupling of H_2 to the W gauge boson is SM-like.

$$Q_W=-\frac{e}{m_W^2}(1+\Delta\kappa+\Delta Q). \tag{22}$$

It is interesting to note that the gauge invariant form (20) is obtained only after adding up the full contributions of a particular sector of any specific model. Gauge invariance along with the cancellation of ultraviolet divergences are thus a test to check the correctness of the result.

In this work we are interested in the contribution to $\Delta\kappa$ and ΔQ from the 3-3-1 model with right-handed neutrinos. As already explained, the exotic quarks do not contribute to the $WW\gamma$ vertex, whereas the extra neutral boson Z' contribution arises from Z-Z' mixing and it is expected to be negligibly small. The only contributions to $\Delta\kappa$ and ΔQ arise from the gauge and scalar sectors. In the former, the static properties of the W boson receive contributions from both the neutral and singly charged bileptons. As far as the scalar

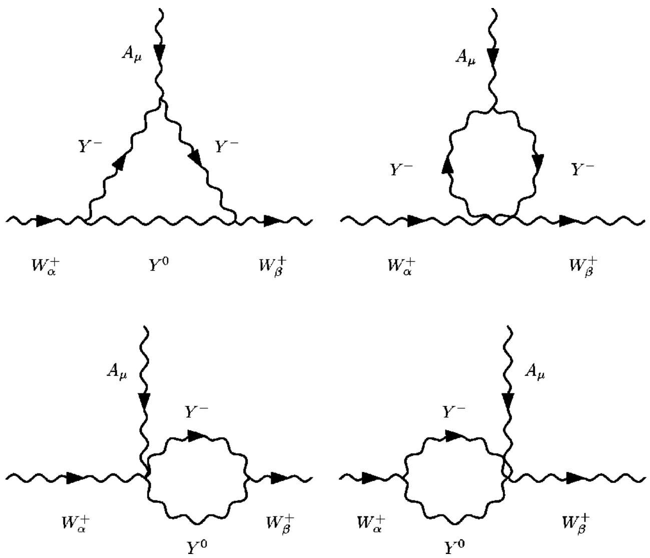

FIG. 3. Feynman diagrams, in the unitary gauge, for the contribution to the on-shell $WW\gamma$ vertex from the gauge sector of the 3-3-1 model with right-handed neutrinos.

sector is concerned, it contributes via the neutral and singly charged Higgs bosons. Before presenting the results for these contributions, we would like to comment briefly on our calculation scheme, which has been already discussed in Refs. [8,18].

Rather than solving the loop integrals by the Feynman parameters technique, one alternative is to use the Passarino-Veltman method [19] to reduce the tensor integrals down to scalar functions. However, this scheme cannot be applied together with the kinematic condition $Q^2=0$ since it requires the inversion of a matrix whose Gram determinant is directly proportional to Q^2. Nevertheless, the Passarino-Veltman reduction scheme can be safely applied for $Q^2\neq 0$, and the limit $Q^2\rightarrow 0$ can be taken at the end of the calculation, which usually requires the application of l'Hôpital rule: $\lim_{Q^2\rightarrow 0} f(Q^2)/Q^2=f'(0)$. This means that the result is given in terms of scalar functions and its derivatives. It was shown in Ref. [18] that any N-point scalar function and its derivatives with respect to any of its arguments can be expressed in terms of a set of $(N-1)$-point scalar functions when the kinematic Gram determinant vanishes. It follows that one can express the three-point scalar function C_0 appearing in the calculation and its derivative with respect to Q^2 in terms of two-point scalar functions B_0. The explicit reduction was presented in Ref. [18]. It is then straightforward to obtain the limit $Q^2\rightarrow 0$. The advantages of this method are twofold: it can be implemented in a computer program [20], which avoids the risk of any error, and the two-point scalar functions can be readily solved analytically or numerically [21]. This calculation scheme is suited to solve loop diagrams carrying vector bosons, which may give rise to some cumbersome tensor integrals.

A. Gauge boson contribution

We turn now to the contributions to ΔQ and $\Delta\kappa$ from the Feynman diagrams of Fig. 3. The amplitudes for these diagrams can be written down with the help of the Feynman rules shown in Fig. 1. After applying the calculation scheme described earlier and taking into account Eq. (20) we are left with

$$\Delta Q^Y = a\left(\frac{12\xi\eta-\chi^2}{2\xi\eta}\right)\left[f_0(\xi,\eta)+f_1(\xi,\eta)\log\left(\frac{\eta}{\xi}\right) + f_2(\xi,\eta)\,\mathrm{arccot}\left(\frac{\xi+\eta-1}{\chi}\right)\right], \tag{23}$$

and

$$\Delta\kappa^Y = \frac{a}{4\xi^2\eta}\left[g_0(\xi,\eta)+g_1(\xi,\eta)\log\left(\frac{\eta}{\xi}\right) + g_2(\xi,\eta)\,\mathrm{arccot}\left(\frac{\xi+\eta-1}{\chi}\right)\right], \tag{24}$$

where we have introduced the definitions $a=g^2/(96\pi^2)$, $\xi=m_{Y^\pm}^2/m_W^2$, $\eta=m_{Y^0}^2/m_W^2$, and $\chi^2=4\xi\eta-(\xi+\eta-1)^2$. The f_i and g_i functions read

$$f_0(\xi,\eta) = -\frac{2}{3}-2(\xi-\eta)^2+3\xi-\eta, \tag{25}$$

$$f_1(\xi,\eta) = [(\eta-\xi)^2-2\xi](\eta-\xi)-\xi, \tag{26}$$

$$f_2(\xi,\eta) = -\frac{2}{\chi}[(\xi-\eta)^4-\eta^3-\xi-\eta(1+\eta)\xi + (3+5\eta)\xi^2-3\xi^3], \tag{27}$$

$$g_0(\xi,\eta) = 9\eta^3+6\eta^4+(\xi-1)^2[1+\xi(7+16\xi)]-\eta^2 \times[35+\xi(59+98\xi)]+\eta\{19+\xi[70+\xi(3+4\xi)]\}, \tag{28}$$

$$g_1(\xi,\eta) = -3\eta^5+3\eta^4(\xi-1)-2\eta(\xi-1)\xi(3+\xi)(3\xi-1) + 2(\xi-1)^3\xi(1+4\xi)+\eta^3[15+\xi(32+49\xi)] - 3\eta^2\{3+\xi[9+\xi(3+17\xi)]\}, \tag{29}$$

$$g_2(\xi,\eta) = 2[3\eta^6-6\eta^5\xi+2(\xi-1)^4\xi(1+4\xi) - 2\eta(\xi-1)^2\xi(2+\xi)(7\xi-1) - 2\eta^4[9+\xi(19+23\xi)] + 2\eta^3[12+\xi(31+45\xi+50\xi^2)] - 3\eta^2(3+\xi\{8+\xi[22+\xi(15\xi-16)]\})]. \tag{30}$$

One interesting scenario is that in which the bileptons are degenerate, i.e., $m_Y^\pm=m_Y^0=m_Y$, which is actually a good assumption for m_{Y^0} much larger than m_W because of the mass splitting (7). In this scenario we obtain

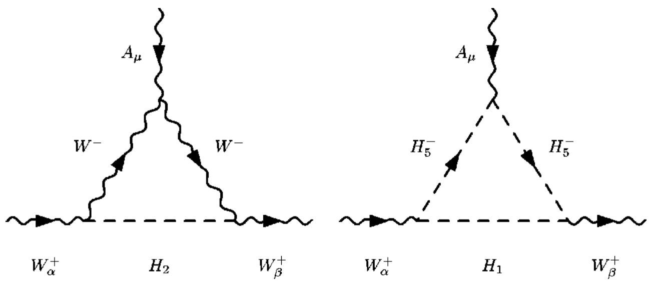

FIG. 4. Feynman diagrams for the scalar contribution to the on-shell $WW\gamma$ vertex in the 3-3-1 model with right-handed neutrinos. Although the H_1 and $H_5^\pm$ scalar bosons also induce three two-point diagrams, they do not contribute to $\Delta\kappa$ or ΔQ. In the scenario described in the text, H_2 coincides with the SM Higgs boson, so the left-hand triangle contribution belongs to the SM and will not be considered a new physics effect.

$$\Delta Q^Y = \frac{a}{\zeta}[1+4\zeta(3\zeta-1)] \times\left[1-\frac{1}{3\zeta}-\frac{2\zeta-1}{\sqrt{4\zeta-1}}\text{arccot}\left(\frac{2\zeta-1}{\sqrt{4\zeta-1}}\right)\right], \quad (31)$$

and

$$\Delta\kappa^Y = \frac{a}{2\zeta^2}\left[12+\zeta[19-36\zeta(1+\zeta)]+\frac{1}{2\zeta} + \frac{(6\zeta-1)\{\zeta[12\zeta(1+\zeta)-7]-2\}}{\sqrt{4\zeta-1}} \times\text{arccot}\left(\frac{2\zeta-1}{\sqrt{4\zeta-1}}\right)\right], \quad (32)$$

with $\zeta = m_Y^2/m_W^2$. It can be shown that in the heavy bilepton limit both (31) and (32) behave as m_W^2/m_Y^2 at the leading order in m_Y. It is then evident that both form factors are insensitive to nondecoupling effects of heavy bileptons.

B. Scalar contribution

In the scenario discussed earlier, the *W* electromagnetic form factors are induced by the charged scalar $H_5^\pm$ and the neutral scalar bosons H_1 and H_2, which give rise to the Feynman diagrams shown in Fig. 4. The Feynman rules necessary for this calculation are shown in Fig. 2. We would like to mention that the neutral H_1 and the charged scalar $H_5^\pm$ bosons induce three additional two-point diagrams, but they are not shown in Fig. 4 as they give no contribution to the electromagnetic form factors. Since the neutral Higgs boson H_2 coincides with the SM one, the contribution from the triangle diagram of the left-hand side of Fig. 4 is in fact a SM effect rather than a new physics effect. The result for this contribution was obtained long ago [4,5]. As for the Feynman diagram of the right-hand side, it yields a contribution similar to that arising in the THDM, as can be inferred from the Feynman rules given in Fig. 2. Although this contribution was already obtained in terms of Feynman-parameter integrals [6], we would like to present an alternative result in terms of elementary functions. The calculation scheme described above yields

$$\Delta Q^H = a\left[\tilde{f}_0(\lambda,\lambda_+)+\tilde{f}_1(\lambda,\lambda_+)\log\left(\frac{\lambda_+}{\lambda}\right) +\tilde{f}_2(\lambda,\lambda_+)\text{arccot}\left(\frac{\lambda+\lambda_+-1}{\tilde{\chi}}\right)\right], \quad (33)$$

and

$$\Delta\kappa^H = a\left[\tilde{g}_0(\lambda,\lambda_+)+\tilde{g}_1(\lambda,\lambda_+)\log\left(\frac{\lambda_+}{\lambda}\right) +\tilde{g}_2(\lambda,\lambda_+)\text{arccot}\left(\frac{\lambda+\lambda_+-1}{\tilde{\chi}}\right)\right], \quad (34)$$

with

$$\tilde{f}_0(\lambda,\lambda_+) = \frac{2}{3}+\lambda_+(2\lambda_+-3)+\lambda-4\lambda_+\lambda+2\lambda^2, \quad (35)$$

$$\tilde{f}_1(\lambda,\lambda_+) = -\lambda_+(\lambda_+-1)^2+\lambda_+\lambda(3\lambda_+-2) -3\lambda_+\lambda^2+\lambda^3, \quad (36)$$

$$\tilde{f}_2(\lambda,\lambda_+) = \frac{2}{\tilde{\chi}}[\lambda_+(\lambda_+-1)^3-\lambda_+\lambda(\lambda_+-1)(4\lambda_+-1) +\lambda_+(6\lambda_+-1)\lambda^2-(1+4\lambda_+)\lambda^3+\lambda^4], \quad (37)$$

$$\tilde{g}_0(\lambda,\lambda_+) = 2-\lambda_++4\lambda, \quad (38)$$

$$\tilde{g}_1(\lambda,\lambda_+) = \frac{1}{2}[1+(\lambda_+-2)\lambda_++\lambda-5\lambda_+\lambda+4\lambda^2], \quad (39)$$

$$\tilde{g}_2(\lambda,\lambda_+) = -\frac{1}{\tilde{\chi}}[(\lambda_+-1)^3-6(\lambda_+-1)\lambda_+\lambda +3(1+3\lambda_+)\lambda^2-4\lambda^3]. \quad (40)$$

In these equations, λ, λ_+, and $\tilde{\chi}$ are given as follows: $\lambda = m_{H_1}^2/m_W^2$, $\lambda_+ = m_{H_5^\pm}^2/m_W^2$, and $\tilde{\chi}^2 = 4\lambda\lambda_+ -(\lambda+\lambda_+-1)^2$.

When both the neutral and the charged Higgs bosons are mass degenerate ($\lambda_+ = \lambda = \tilde{\zeta}$) Eqs. (33) and (34) yield

$$\Delta Q^H = \frac{2a}{3}\left[1-3\tilde{\zeta}+\frac{3\tilde{\zeta}(2\tilde{\zeta}-1)}{\sqrt{4\tilde{\zeta}-1}}\operatorname{arccot}\left(\frac{2\tilde{\zeta}-1}{\sqrt{4\tilde{\zeta}-1}}\right)\right] \tag{41}$$

and

$$\Delta\kappa^H = a\left[2+3\tilde{\zeta}+\frac{[1-3\tilde{\zeta}(1+2\tilde{\zeta})]}{\sqrt{4\tilde{\zeta}-1}}\operatorname{arccot}\left(\frac{2\tilde{\zeta}-1}{\sqrt{4\tilde{\zeta}-1}}\right)\right]. \tag{42}$$

We have numerically evaluated the above results and found agreement with those obtained in Ref. [6].

IV. NUMERICAL EVALUATION

We will now analyze the behavior of the form factors for some range of values of the bilepton and the Higgs boson masses. These are the only free parameters which enter into ΔQ and $\Delta\kappa$. We will analyze separately each contribution.

A. Gauge boson contribution

To begin with, it is worth analyzing the current bounds on the bilepton masses from both theoretical and experimental grounds. First of all, it is interesting to note that the matching of the gauge couplings constants at the $SU_L(3)\times U_X(1)$ breaking leads to $4\sin\theta_W \leqslant 1$ in the minimal 3-3-1 model [10], from which an upper bound on the bilepton masses can be derived, namely $m_Y \lesssim 1$ TeV. Therefore, this model would be confirmed or ruled out by collider experiments in a near future. The version with right-handed neutrinos requires, however, that $4\sin\theta_W \leqslant 3$, which yields no useful constraint on m_Y. As already mentioned, because of the symmetry-breaking hierarchy, the splitting $|m_{Y^\pm}^2 - m_{Y^0}^2|$ is bounded by the W boson mass. Therefore m_{Y^0} and $m_{Y^\pm}$ are not arbitrary at all. One cannot, for instance, make large $m_{Y^\pm}$ while keeping fixed m_{Y^0} or vice versa. In fact, when $m_{Y^0} \gg m_W$, the charged and neutral bileptons would become degenerate. As far as the lower bounds on the bilepton masses are concerned, in Ref. [12] it was argued that the data from neutrino neutral current elastic scattering give a lower bound on the mass of the new neutral gauge boson m_{Z_2} in the range of 300 GeV, which along with the symmetry-breaking hierarchy yield $m_{Y^\pm} \sim m_{Y_0} \sim 0.72 m_{Z_2} \geqslant 220$ GeV. A similar bound was obtained in Ref. [14] from the observed limit on the "wrong" muon decay $R=\Gamma(\mu^- \to e^- \nu_e \overline{\nu_\mu})/\Gamma(\mu^- \to e^- \overline{\nu_e} \nu_\mu) \leqslant 1.2\%$, which leads to $m_{Y^\pm} \geqslant 230 \pm 17$ GeV at 90% confidence limit (C.L.). These lower bounds on $m_Y^\pm$ are in agreement with that obtained from the latest BNL measurement on the muon anomaly [12,15].

According to the above discussion, we deem it interesting to evaluate the form factors in the range 100 GeV $\leqslant m_{Y^0} \leqslant$ 1000 GeV, which will be useful to illustrate their behavior and get an idea about their size. At this point it is important to mention that to cross-check our results, the form factors were obtained independently by the Feynman parameters method. The integrals were evaluated numerically and the result was compared with the one obtained by the Passarino-Veltman method. A perfect agreement was observed. We refrain from presenting the results in terms of parametric integrals since the closed expressions (23) and (24) can be handled more easily.

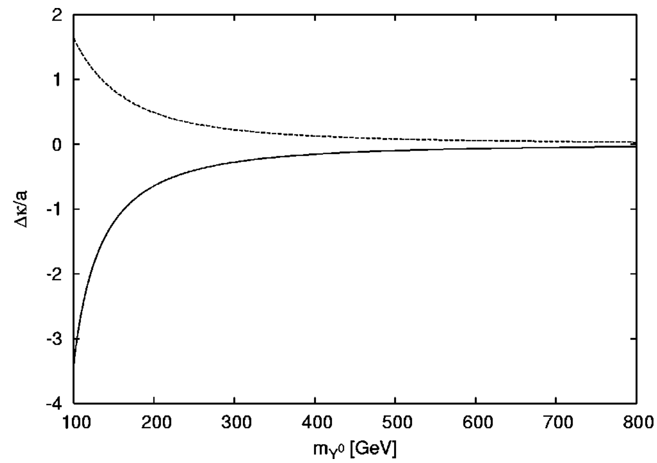

FIG. 5. Gauge boson contribution to $\Delta\kappa$ in the 3-3-1 model with right-handed neutrinos as a function of the mass of the neutral bilepton when the charged bilepton mass is maximal (solid line) and minimal (dashed line). According to the mass splitting, the extremal values are given by $m_{Y^\pm}^2 = m_{Y^0}^2 \mp m_W^2$. The form factor is restricted to lie in the area enclosed by the lines.

The $\Delta\kappa^Y$ and ΔQ^Y form factors are shown in Figs. 5 and 6 as a function of the neutral bilepton mass. There are two curves in each plot, which correspond to the extremal values of $m_{Y^\pm}$, namely $m_{Y^\pm}^2 = m_{Y^0}^2 - m_W^2$ and $m_{Y^\pm}^2 = m_{Y^0}^2 + m_W^2$. The form factors are restricted to lie in the area surrounded by the two extremal lines. In Fig. 5 it is clear that the bileptons can give a negative or positive contribution to $\Delta\kappa^Y$, which depends on which bilepton is the heaviest. Also, we can observe that $\Delta\kappa^Y$ is sensitive to the value of the splitting and has a larger size for nondegenerate bileptons than for degenerate bileptons. The $\Delta\kappa^Y$ form factor in the latter scenario is displayed in Fig. 7. In this plot we can observe that, when one of the bilepton masses is close to m_W and the splitting is maximal, $\Delta\kappa^Y$ can have a size of about one order of magni-

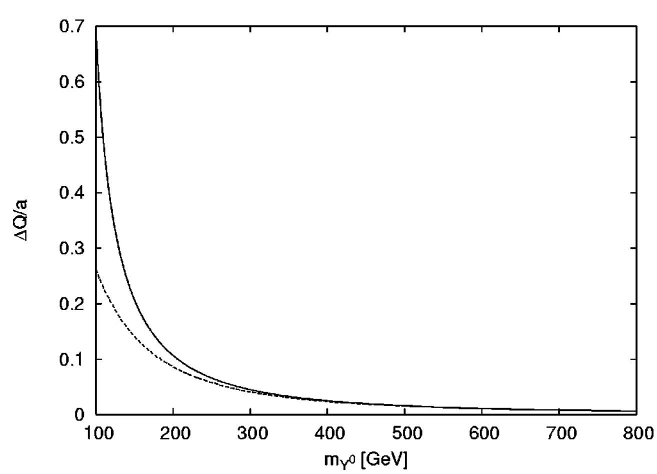

FIG. 6. The same as in Fig. 5 for the ΔQ form factor.

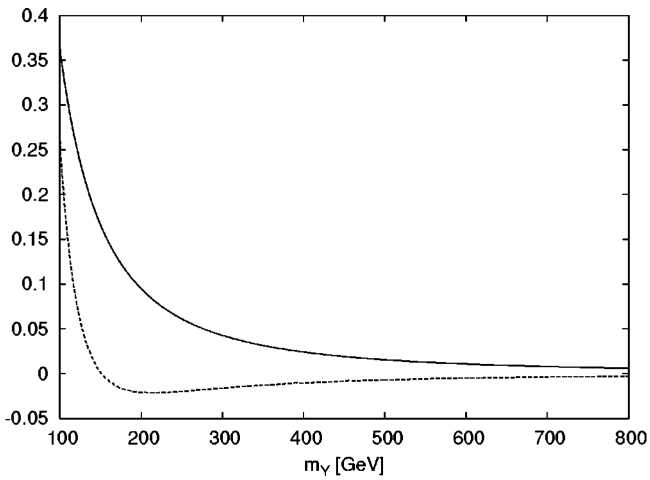

FIG. 7. Bilepton contribution to the $\Delta\kappa$ (solid line) and ΔQ (dashed line) form factors, in units of *a*, in the 3-3-1 model with right-handed neutrinos when the bileptons are degenerate and have a mass m_Y.

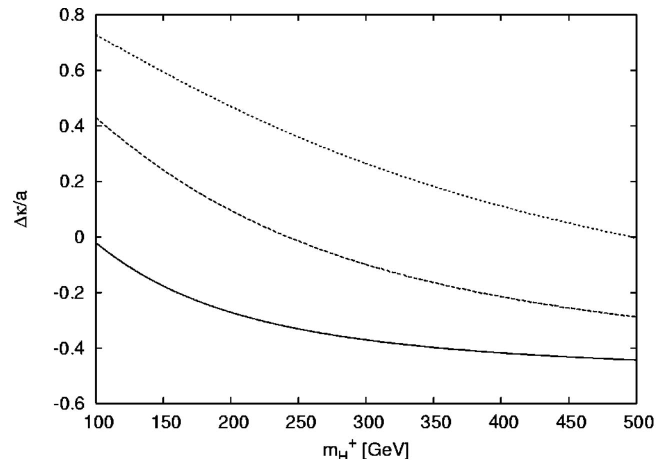

FIG. 8. Scalar contribution to $\Delta\kappa$ in the 3-3-1 model with right-handed neutrinos as a function of $m_{H_5^\pm}$ for different values of the mass of the neutral Higgs boson m_{H_1}: 115 GeV (solid line), 250 GeV (dashed line), and 500 GeV (dashed-dotted line).

tude above than the one obtained when there is degeneracy of the bilepton masses. On the contrary, ΔQ^Y is less sensitive to the mass splitting and the extremal values of $m_{Y^\pm}$ yield values of the same order of magnitude than the one observed in the degenerate case, which is also shown in Fig. 7. From these plots we can conclude that the size of the contribution to the form factors from the gauge sector of the 3-3-1 model with right-handed neutrinos is about of the same order of magnitude than the one obtained in the case of the bilepton contribution in the minimal 3-3-1 model and in the case of the contributions of other SM extensions. The larger absolute values are obtained for lighter bileptons and when the charged bilepton mass reaches its maximal allowed value. It is interesting to note that all weakly coupled theories studied up to now give a contribution to the *W* form factors of similar size [6–8].

In Fig. 7, we can clearly see that both $\Delta\kappa^Y$ and ΔQ^Y are insensitive to heavy physics effects and approach zero very quickly as the bilepton masses increase. The only scenario which may give rise to nondecoupling effects is that in which one bilepton mass is kept fixed while the other is made very large, which of course is forbidden by the mass splitting constraint (7). In Ref. [8] we already discussed a similar situation arising in the minimal 3-3-1 model, with a doubly charged bilepton playing the role of the neutral one. This case also resembles the one discussed in Ref. [22] for a scalar doublet which acquires mass from a bare parameter. The reason why there is no decoupling effects is not surprising since a large bilepton mass implies a large VEV which is heavier than the electroweak scale. On the contrary, the splitting between the bilepton masses arises from VEV's which are of the size of the electroweak scale. This is to be contrasted with the case of a fermion pair accommodated in a $SU_L(2)$ doublet, which are known to give rise to nondecoupling effects. Since the fermions acquire their masses from Yukawa couplings, a large fermion mass implies a large coupling, whereas a heavy bilepton mass implies a large VEV instead of a large coupling. The former scenario is the one which is known to break down the decoupling theorem [23]. Very interestingly, even in those scenarios in which $\Delta\kappa^Y$ is sensitive to heavy physics effects, ΔQ^Y is not [3]. The decoupling theorem establishes that only those terms arising from renormalizable operators may be sensitive to nondecoupling effects, whereas those terms induced by nonrenormalizable operators are suppressed by inverse powers of the heavy mass [23]. Thus ΔQ^Y always decouples when one particle circulating in the loop is made large since it is generated by a nonrenormalizable dimension-six operator, but $\Delta\kappa^Y$ may be sensitive to nondecoupling effects as it is induced by a dimension-four operator.

B. Scalar contribution

The scalar contribution to the $\Delta\kappa$ and ΔQ form factors is shown in Figs. 8 and 9 as a function of the charged Higgs boson mass and for different values of the neutral Higgs boson mass. We would like to emphasize that the values shown in those plots correspond to the contribution from

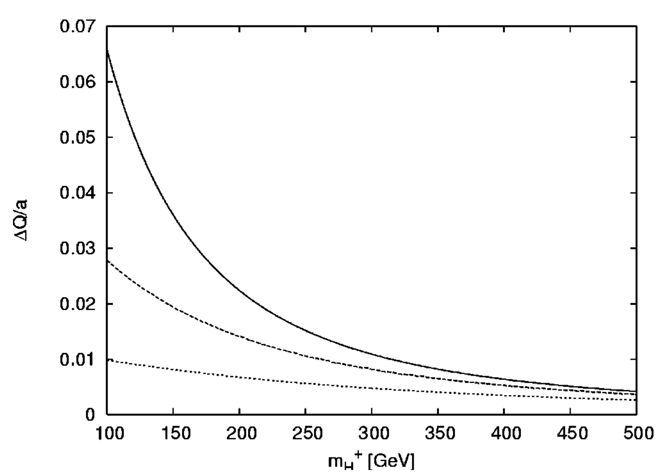

FIG. 9. The same as in Fig. 5 for the ΔQ form factor.

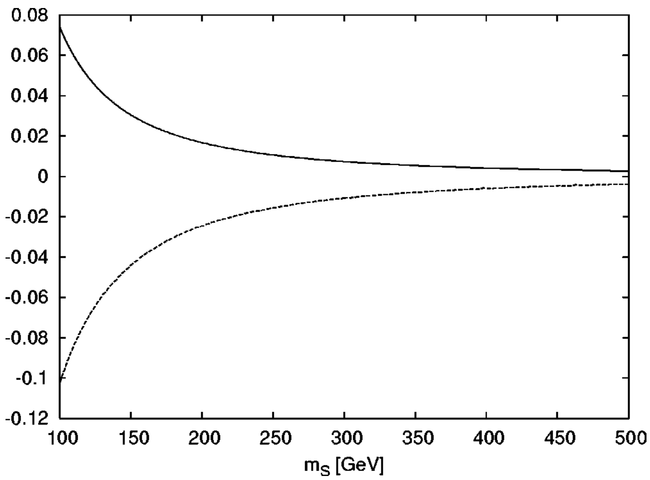

FIG. 10. Scalar contribution to the $\Delta\kappa$ (solid line) and ΔQ (dashed line) form factors, in units of a, in the 3-3-1 model with right-handed neutrinos when H_1 and $H_5^{\pm}$ are degenerate and have a mass m_S.

new physics only. From these figures, we can observe that both $\Delta\kappa^H$ and ΔQ^H decrease rapidly for increasing $m_{H_5^{\pm}}$. In fact, the latter goes to zero quickly as either m_{H_1} or $m_{H_5^{\pm}}$ increase. Although $\Delta\kappa^H$ seems to increase with increasing m_{H_1} for a relatively light $m_{H_5^{\pm}}$, it approaches the limiting value $\Delta\kappa^H = a$ for very large m_{H_1}. We can also observe that when the scalar boson masses are of the same size than those of the bilepton gauge bosons, the contribution from the Higgs sector is about one order of magnitude below than that of the gauge sector. In fact, if the scalar boson masses are degenerate, the respective contribution to $\Delta\kappa$ and ΔQ is very small, as shown in Fig. 10. As pointed out in Ref. [6], this reflects the fact that the Higgs boson is not strongly interacting. Thus, for the Higgs sector to give a large correction to the W form factors, it would be necessary to have the contributions from an unrealistic number of Higgs bosons. Although we are restricted to a particular form of the scalar potential, we can conclude that we cannot expect large contributions from this sector even in the most general case.

It is interesting to analyze the behavior of Eqs. (33) and (34) in the decoupling limit. It turns out that ΔQ^H always vanishes no matter which one of m_{H_1} or $m_{H_5^{\pm}}$ is made large. On the other hand, $\Delta\kappa^H$ do may give rise to nondecoupling effects. If both m_{H_1} and $m_{H_5^{\pm}}$ become simultaneously large, $\Delta\kappa^H$ vanishes, but when m_{H_1} becomes infinite and $m_{H_5^{\pm}}$ remains finite, it approaches the constant value $\Delta\kappa^H = a$; when the situation is reversed, $\Delta\kappa^H \to -a/2$. This is in accordance the previous discussion on the decoupling properties of the W form factors.

Finally, we would like to compare the size of the new contributions with those of the SM, which is known to give the following one-loop corrections to $\Delta\kappa$ and ΔQ [4]: $\Delta\kappa^{SM}_{max} \approx 30\, a$ and $\Delta Q^{SM}_{max} \approx 5\, a$. The contribution from the 3-3-1 model with right-handed neutrinos is thus only a few percent that of the SM. From all the studies presented in the literature [7], it can be inferred that only those models in which there are contributions from a large number of particles would have the chance of giving large corrections to the W form factors.

V. FINAL REMARKS

In this work we have calculated the static quantities of the W boson in the framework of the 3-3-1 model with right-handed neutrinos. Apart from the usual SM contributions, there are new contributions from the gauge and the scalar sectors. In the former there is a new contribution induced by a singly charged $Y^{\pm}$ and a complex neutral gauge boson Y^0, called bileptons. In the scalar sector there is the contribution from a singly charged Higgs boson $H_5^{\pm}$ and two neutral scalar bosons H_1 and H_2, but H_2 coincides with the SM Higgs boson and its contribution should be identified with a SM effect rather than with new physics. Although the model predicts three exotic quarks and an extra neutral gauge boson Z', these particles give no contribution to ΔQ and $\Delta\kappa$. It turns out that the exotic quarks do not couple to the W boson as they are $SU_L(2)$ singlets, whereas Z' can only contribute through Z-Z' mixing and its contribution is expected to be negligibly small. Analytical expressions were presented for both nondegenerate and degenerate masses of the bileptons and the Higgs bosons. The loop integrals were worked out by a modified version of the Passarino-Veltman reduction scheme. To cross-check our results, the form factors were obtained independently by the Feynman parameter technique and the resulting integrals were numerically evaluated and compared with the results obtained through the Passarino-Veltman method. It was found that the new contributions can be of the same order of magnitude as those arising in other weakly coupled renormalizable theories. It is interesting to note that the contribution from the scalar sector is similar to that of a THDM. This means that the form factors will not help us to discriminate between different theories. Instead the on-shell $WW\gamma$ vertex would be useful to test the particular theory realized in nature with high precision once all the free parameters of the theory are known. In the scenario in which the non-SM particles circulating in the loops (bileptons or Higgs bosons) are degenerate, the form factors are smaller than in the case in which they are nondegenerate. It was also found that in the scenario in which the bilepton and scalar boson masses are of the same order of magnitude, the gauge sector gives dominant contribution to the W form factors. The nondecoupling properties of the $\Delta\kappa$ and ΔQ form factors were analyzed. It was found that ΔQ is always of decoupling nature, whereas $\Delta\kappa$ is sensitive to heavy Higgs bosons but insensitive to heavy bileptons. In fact, the numerical analysis shows that the contribution from a heavy bilepton with mass of the order of 1 TeV is negligibly small.

ACKNOWLEDGMENT

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